

NEW REGIME FOR DENSE STRING NETWORKS^a

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We uncover a new transient regime that reconciles the apparent inconsistency of the Martins Shellard one scale damped string evolution model¹ with the initial conditions predicted by the Kibble mechanism for string formation in a second order phase transition. This regime carries (in a short cosmic time $\sim .1t_c$) the dense string network created by the Kibble *mechanism* to the (dilute) Kibble *regime* in which friction dominated strings remain till times $t_* \sim (M_P/T_c)^2 t_c$. This is possible because the cosmic time at the phase transition (t_c) is much larger than the damping time scale $l_f \sim T_c^2/T^3$. Our result has drastic implications for various non-GUT scale string mediated mechanisms.

1 Introduction

Topological strings are an inevitable consequence of symmetry breaking phase transitions when the first homotopy group of the vacuum manifold is non trivial. They can trap gauge flux and could have have important consequences for the structure of the present day universe².

Most research has focussed on strings formed at temperatures above GUT scales (which are relevant for galaxy formation. Recently mechanisms for baryogenesis, neutralino production etc³ based on dense string networks formed in second order phase transitions at scales far below the GUT scale have also been proposed. Such networks would be friction dominated till very low temperatures $t_* \sim (M_P/T_c)^2 t_c$. An estimate of how the network correlation length L evolves between is a critical input for such models.

Due to Everett and Aharanov-Bohm scattering² a string with velocity \vec{v} relative to the ambient plasma feels a velocity dependent frictional force characterized by a lengthscale $l_f = \eta^2/\beta T^3$ (β is a numerical parameter $\sim 10^{-2}-1$). String curvature causes continual motion so energy conservation implies loss of string length due to frictional damping. For non-GUT strings this is the dominant cause of dissipation of string length over much of the network's history. Thus any damped network evolution model must account for the average string velocity v . Such a simple one scale model which tracks both L and v was given by Martins and Shellard(MS)¹. Their equations read :

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$$\begin{aligned}\frac{dL}{dt} &= HL(1+v^2) + \frac{Lv^2}{2l_f} + \frac{cv}{2}, \\ \frac{dv}{dt} &= (1-v^2) \left[\frac{k}{L} - v \left(2H + \frac{1}{l_f} \right) \right],\end{aligned}\quad (1)$$

where H is the Hubble parameter, k characterizes the “wiggleness” and c the self intersection or “chopping” rate. k is ~ 1 in the frictional era but smaller at the late times of the linear scaling regime when the strings are undamped and therefore kinky. For dense networks reconnection of loops back may be important leading to a lower effective value of c but geometrical estimates give $c \sim 1$.

It is important that $l_f(t_c) \sim \eta^{-1} \ll t_c \sim M_P/\eta^2$ when ($\eta \ll M_P$). Thus a short interval of cosmic time is very long in units of the frictional length scale. *This simple observation is at the root of the new regime proposed in this paper.*

In the MS model the network curvature radius $R \equiv L$. So the curvature force is never zero which forces the string to move and dissipate its rest energy. One may visualize the growth of L and R as being due to the decaying away of curved and looped regions with high curvature leaving behind a network with less string (larger L) and lower curvature (larger R).

Clearly the initial correlation length L_i must obey $\eta^{-1} < L_i < t_i \sim \sigma/\eta$ where $\sigma \sim M_P/\eta$. These limits correspond to those expected in a second and first order transition respectively, while on energetic grounds one expects $v_i \sim 1$.

The Kibble mechanism² predicts ϵ_K string lengths of size $1/T_c$ are formed in each correlation volume $\sim T_c^{-3}$ (where $\epsilon_K \sim 10^{-1} - 10^{-3}$) so $L_i \sim \epsilon_K^{-1/2}/T_c$.

MS consider only the range $\sqrt{\sigma}/\eta < L_i < \sigma/\eta$ corresponding to initially non-dense networks. They found that the phase transition is followed by

i) *Stretching Regime*: it occurs only when $L_i > \sqrt{\sigma}/\eta$. As $L_i \rightarrow \sqrt{\sigma}/\eta$ the duration of the stretching regime goes to zero. L, v scale as :

$$L_S = L_i \left(\frac{t}{t_c} \right)^{1/2} \quad v_S = \frac{t}{\sigma L_i} \quad (2)$$

ii) The *Kibble Regime*⁴ follows the *stretching* regime. It is a terminal velocity regime arising from the balance of the frictional and curvature forces.

$$L_K = \left(\frac{2k(k+c)}{3\sigma} \right)^{1/2} (t)^{5/4} t_c^{-1/4}; \quad v_K = \left(\frac{3k}{2(k+c)\sigma} \right)^{1/2} \left(\frac{t}{t_c} \right)^{1/4} \quad (3)$$

The Kibble regime begins at $t_K \sim L_i^{4/3} M_P^{1/3}$, *i.e. earlier for lower L_i* . Since $t_K \geq t_c$ for consistency it appears as if $L_i \geq \sqrt{\sigma}/\eta$ necessarily ! Now $L_K(t_c) \sim \sqrt{\sigma}/\eta$ while a second order phase transition predicts $L_i \sim 1/\eta$. There is a wide mismatch between the two values so it appears the MS model leads one to the contradictory conclusion that although the Kibble regime begins immediately after the phase transition yet its density must necessarily be much lower than that predicted by the Kibble mechanism . A similar mismatch holds for v .

The resolution lies in the observation that $l_f(t_c) \ll t_c$. The correct time scale to use when $t \sim t_c$ is $l_f \sim 1/\eta$ not t_c . Consider then the evolution of L and v in the period immediately following the phase transition *i.e* when $\hat{t} = t - t'_c \ll t'_c$, where $t'_c \sim 10t_c$ (say). The precise value of $t'_c \sim t_c$ (at which the evolution of the one scale model is taken to begin) is unimportant as can be checked by varying t'_c by a factor of 10. We define a dimensionless time $x = \eta \hat{t}$ and network scale $l = \eta L$. Note that this amounts to measuring time in units of the damping time scale and avoids the introduction of large numbers in the evolution equations which result if one uses t_c as the unit of time. Then the condition $\hat{t} = t - t'_c \ll t_c$ translates to $x \ll \sigma$. Even for GUT scale strings σ is $\sim 10^4$ so the variable x can change by many orders of magnitude before the approximation becomes invalid. Expanding the terms in the evolution equation around t'_c one finds that as long as $l < \sigma^{1/2}$ and $v > \sigma^{-1/2}$ the Hubble term is completely dominated by the velocity dependent terms. To leading order we have

$$2 \frac{dl}{dx} = lv^2\beta + cv ; \quad \frac{dv}{dx} = (1 - v^2) \left(\frac{k}{l} - v\beta \right). \quad (4)$$

We must find the behaviour of l and v for large x beginning from natural initial conditions (Kibble Mechanism) where $l_i \sim 1$. l always increases while v will increase when $k \geq v\beta l$ and vice versa. Regardless of v_i the evolution locks on to a trajectory where v is very closely approximated by $k/\beta l$. Then for $\sigma \gg x \gg 1$ one gets

$$l = \left(\frac{(k+c)k}{\beta} \right)^{1/2} x^{1/2} ; \quad v = \left(\frac{k}{\beta(k+c)} \right)^{1/2} x^{-1/2}. \quad (5)$$

This regime persists until the growth of the Hubble term and the decrease of the velocity dependent term makes them comparable *i.e* till $x \sim 10^{-1}\sigma$ (for typical values of k, c) by when $t \sim t'_c + .1t_c$. After this the effect of the Hubble terms causes a shift in the power-law exponents and the Kibble regime begins. Our new regime makes it possible to extend the applicability of the MS model into the regime of initially dense networks where it appeared to give

a contradiction. When it ends one has $L \sim \sqrt{\sigma}\eta^{-1}, v \sim \sigma^{-1/2}$, which are just right to match the Kibble regime beginning at $t \sim t'_c$.

The most remarkable feature of this regime is thus that *the values of L and v very shortly after the phase transition are essentially independent of their initial values*. This is in sharp contrast to the “stretching regime” $L \sim L_i(t/t_c)^{1/2}, v \sim t(L_i\sigma)^{-1}$ found for initial conditions $L_i > \sigma^{1/2}/\eta$.

The Lyapunov exponents obtained by linearizing the eqns. (4) around the solutions eqns. (5) are $\sim -c/2(k+c)x, -\beta$ i.e negative so that our regime is an attractor as x increases. We have also verified the above asymptotic analysis given by numerical integration. For large x the asymptotic solutions given in eqn.(5) are in excellent agreement with the results of numerical integration of eqns.(4) irrespective of whether the initial velocities are relativistic or very small.

Similar conclusions hold for string length in loops at the time of formation or that in loops that form by self-intersection. However superconducting loops (vortons) may evade the effects of curvature driven damping since the supercurrent on vortons can stabilize their radius.

To summarize : The one scale model with friction and velocity evolution¹ leads to a consistent picture of network evolution during the entire friction dominated era. It predicts that *the dense, fast network formed in a second order phase transition crosses over into a sparse slow network in a time $\sim .1t_c$* . Thus mechanisms³ that rely on the high density of string length predicted by the Kibble mechanism are unlikely to work unless string loops are stabilized against curvature driven contraction, e.g by superconducting currents. Our analysis shows that friction can be a very important mechanism for string decay. Further analysis of the microscopic mechanism of frictional conversion of the coherent string condensate into plasma energy is required.

References

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